The Development of a Semi-Empirical Analytical Model for the Modal Propellant Gauge Project

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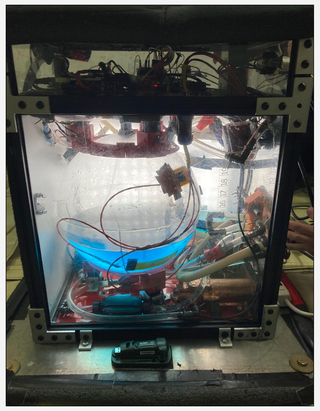
INTRODUCTION

- Modal Propellant Gauging (MPG) uses structural acoustic resonant frequencies to determine liquid mass in a tank
- MPG-Propellant Refueling and on-Orbit Transfer Operations (MPG-PROTO) measures liquid mass levels while transferring liquid between tanks

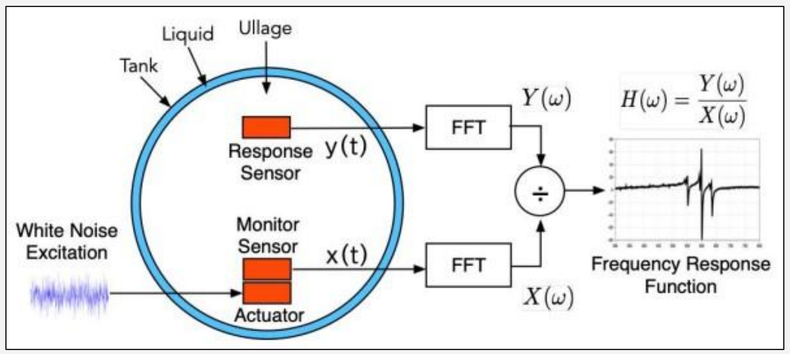


OBJECTIVES

1. Determining gauging resolution for a microgravity drain representative of an in-space propellant transfer
2. Develop and determine the accuracy of an analytical model that predicts fill fraction from mode frequency

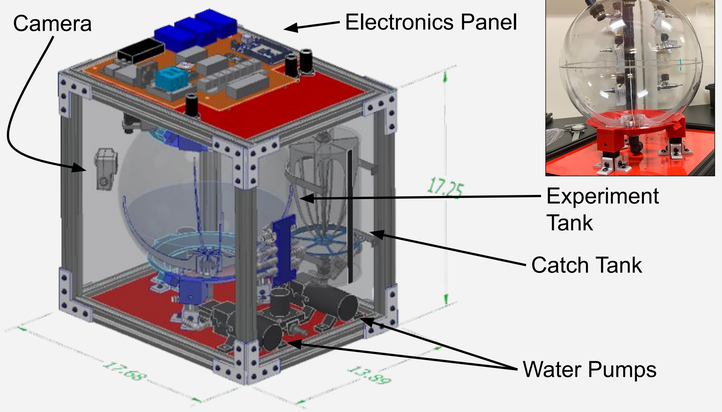
MODAL PROPELLANT GAUGING

- White noise vibrates the tank through the Actuator PZT patch and is measured by Sensor and Monitor patches
- The Sensor and Monitor are Fourier transformed to obtain a Frequency Response Function (FRF) of the tank
- The experimental FRF is correlated to a library of known FRF data to determine the liquid mass in the tank



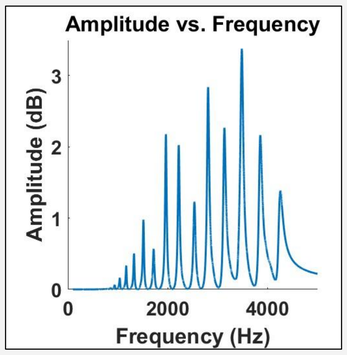
EXPERIMENT DESIGN

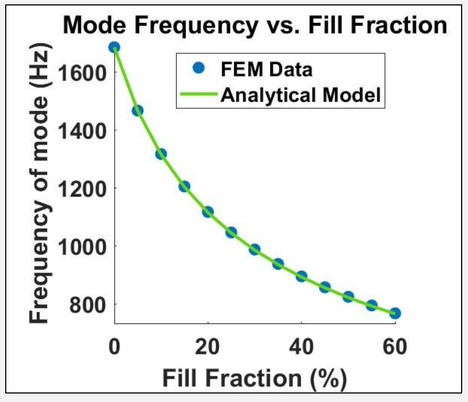
- Liquid is transferred between the experiment and catch tanks to different fill levels



FINITE ELEMENT MODEL

- A Finite Element Model (FEM) is a computational model of the physical structure used to predict resonant modes
- Resonant modes appear in the frequency domain as amplitude peaks
- As liquid mass increases, the resonant mode frequencies decrease





FEM data for a 35% fill fraction. Each peak is a different resonant mode. The experimental FRF generates similar data.

The frequency of the lowest mode for all of the fill fractions in the FEM. An analytical model, which uses fill fraction to predict frequency, is also shown.

SEMI-EMPIRICAL ANALYTICAL MODEL

- Resonant modes are constrained by the structure's geometry, so the analytical model is derived from the tank's geometry
- Model originally solved for a partially full cylindrical tank [1]

$$f = \frac{f_0}{\sqrt{1+\frac{\rho_l a}{\rho_s h_s} M^*}}$$

The cylindrical tank model

$$f = \frac{f_0}{\sqrt{1+Ar^b}}$$

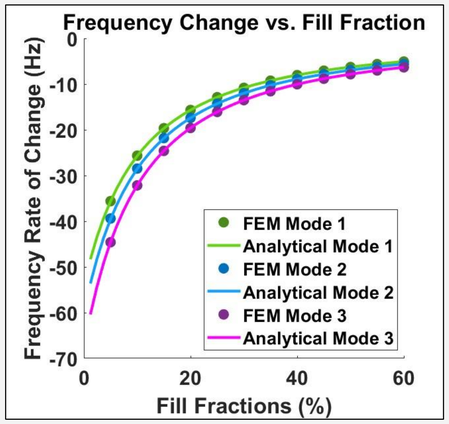
The generalized model

f: mode frequency
f_0: empty tank frequency
ρ_l: liquid density
a: radius
M*: modal mass
ρ_s: tank density
h_s: tank thickness

The analytical model

$$f = \frac{f_0}{\sqrt{1+Ar}}$$

- Generalized as fill fraction (r), and the parameters (A,b) that optimize the model's fit to the FEM [2]
- For a sphere, b=1
- The model predicts fill fraction from resonant frequency
- Frequency rate of change is the modal frequency fill fraction sensitivity and the derivative of the analytical model frequency with respect to fill fraction



The frequency shift of the three lowest modes for the FEM and the analytical model.

Mode frequency shift [2]

$$\frac{\partial f}{\partial r} = -\frac{1}{2r} f\left[1 - \left(\frac{f}{f_0}\right)^2\right]$$

CONCLUSIONS

- The analytical model was fit to the FEM with a R^2 near 1
- The high accuracy fit of the analytical model supports a one-parameter model to predict mode frequencies as a function of fill fraction

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[1] W. H. Chu, Breathing vibrations of a partially filled cylindrical tank—linear theory, Journal of Applied Mechanics 30, 532 (1963).
[2] K. Crosby, E. A. Hurlbert, and R. J. Werlink, Liquid mass gauging during propellant transfer in microgravity, Acta Astronautica (2024)